3.1 Rational Function Properties and Graphs MATH 161 THOMPSON

1) State whether the following statement is true or false.

The quotient of two polynomial expressions is a rational expression.

Choose the correct answer below.

- False
- True

A rational function is a function of the form $R(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions and q is not the zero polynomial.

2) Determine whether the statement below is true or false.

The domain of every rational function is the set of all real numbers.

Choose the correct answer below.

- True
- False

The domain of a rational function is the set of all real numbers except those for which the denominator is 0.

3) Determine whether the following statement is true or false.

The graph of a rational function may intersect a horizontal asymptote.

Choose the correct answer below.

- True
- False

Both horizontal and oblique asymptotes describe the end behavior of a rational function. The graph of a function may intersect a horizontal or oblique asymptote, but it will never intersect a vertical asymptote.

4) Decide whether the following statement is true or false.

The graph of a rational function may intersect a vertical asymptote.

Choose the correct answer below.

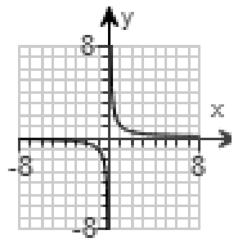
- False
- True

The graph of a function will never intersect a vertical asymptote. Note that the graph of a function may intersect a horizontal asymptote.

5) For a rational function R, if the degree of the numerator is less than the degree of the denominator, then R is proper.

For a rational function R, if the degree of the numerator is less than the degree of the denominator, then R is proper. Note that if the degree of the numerator is greater than the degree of the denominator, then R is improper.

6) Graph $y = \frac{1}{x}$.



7) If a rational function is proper, then $y = 0$ is a horizontal asymptote.

When a rational function R(x) is proper, the degree of the numerator is less than the degree of the denominator; as $x \to -\infty$ or as $x \to \infty$, the value of R(x) approaches 0.

8) Find the domain of the following rational function.

$$R(x) = \frac{2x}{x + 12}$$

Cannot have zero in denominator

set equal to zero $x + 12 \neq 0$

Select the correct choice below and, if necessary, your choice.

A. The domain of R(x) is $\{x \mid x \neq -12\}$.

9) Find the domain of the following rational function.

$$H(x) = \frac{-3x^2}{(x-5)(x+3)}$$

Cannot have zero in denominator

set equal to zero $x - 5 \neq 0$ and $x + 3 \neq 0$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The domain of H(x) is $\{x \mid x \neq 5, -3\}$.

10) $F(x) = \frac{5x(x-5)}{6x^2 - 41x - 7}$ *Factor denominator*

slide and divide $x^2 - 41x - 42$ *factors of 42 that subtract to give you 41*

$\frac{(x-42)}{6}\frac{(x+1)}{6}$ then divide by 6 and reduce

$x \neq 7, -\frac{1}{6}$

11) Find the domain of the following rational function.

$$H(x) = \frac{12x^2 + x}{x^2 + 9}$$

You cannot factor denominator with a plus sign

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The domain of H(x) is {x | }.
(Type an inequality. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

B. The domain of H(x) is the set of all real numbers.

12) Find the domain of the following rational function.

$$R(x) = \frac{4(x^2 - 4x - 60)}{5(x^2 - 100)}$$

Factor denominator

Difference of two squares

(x+10)(x-10

$x \neq -10, 10$

Select the correct choice below and fill in any answer boxes within your choice.

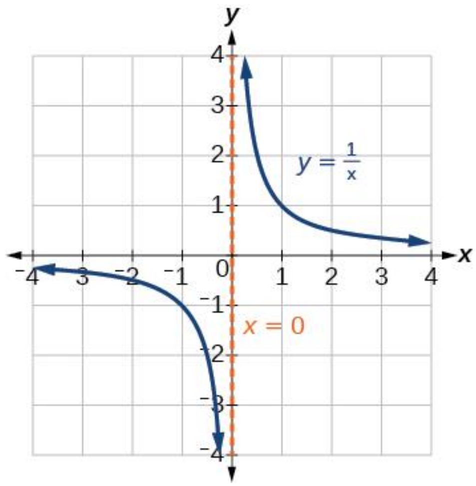
A. The domain of R(x) is $\{x \mid x \neq -10, 10\}$.

GRAPH OF RECIPROCAL FUNCTION

f(x) = $\frac{1}{x}$

Asymptotes are
x = 0 and y = 0

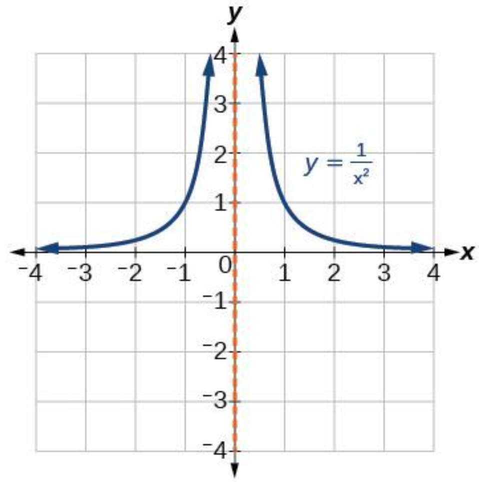
Domain: {x|x≠0} Range: {y|y≠0}



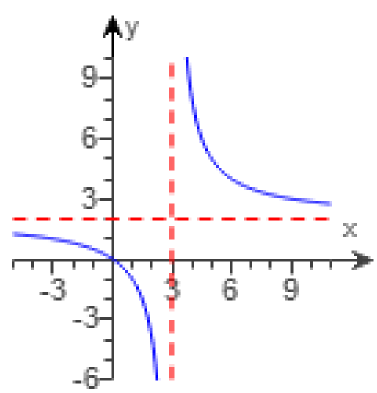
f(x) = $\frac{1}{x^2}$

Asymptotes are
x = 0 and y = 0

Domain: {x|x≠0} Range: {y|y>0}



13) Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any

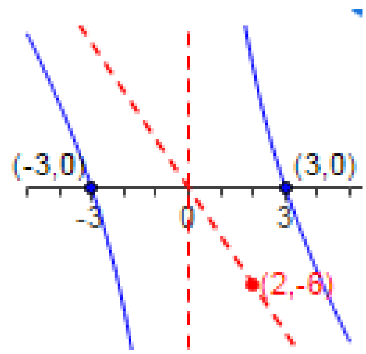


a) D: x ≠ 3   *red vertical line
R: y ≠ 2   *red horizontal line

b) x = 0   *graph crosses x-axis
y = 0   *graph crosses y-axis

c) y = 2   *equation of the red horizontal line
d) x = 3   *equation of the red vertical line
e) No oblique asymptote

14) Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any



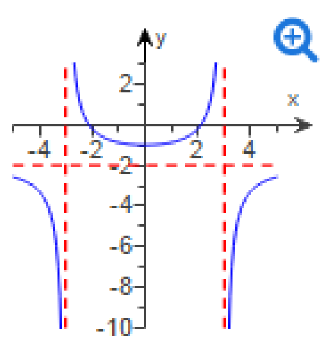
a) D: x≠ 0 *red vertical line
R: all real numbers *NO horizontal line

b) x = -3,3 *graph crosses x-axis
no y intercepts *graph crosses y-axis

c) no horizontal asymptotes *NO horizontal line
d) x = 0 *equation of the red vertical line
e) Oblique asymptote *equation of the diagonal line
y = -3x *(use slope ($\frac{-6}{2}$) and y-intercept = 0)*

15)

Use the graph shown to find the following.
(a) The domain and range of the function
(b) The intercepts, if any
(c) Horizontal asymptotes, if any
(d) Vertical asymptotes, if any
(e) Oblique asymptotes, if any



a) D: x≠ -3,3 *two red vertical lines
R: y < -2, y ≥ -1 *the area excluding -1 to -2

***NOTE* x intercepts may be -1.4, 1-4 if they look between* 1 and 2**

b) x = -2, 2 *graph crosses x-axis
y = -1 *graph crosses y-axis

c) y = -2 *equation of the red horizontal line
d) x = -3,3 *equation of the red vertical lines
e) no oblique asymptote

*GENERAL RULES ON DOMAIN AND RANGE

Graph $y = \frac{1}{x}$.

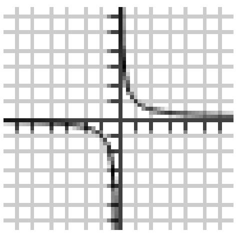
$\boxed{F(x) = 8 + \frac{1}{x},}$

Up 8 units, the range changes

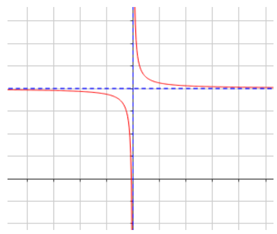
Graph $y = \frac{1}{x^2}$

$\boxed{F(x) = \frac{1}{(x+8)^2}}$

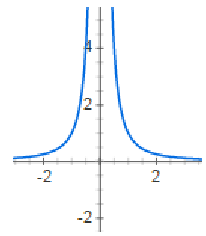
left 8 units, domain changes



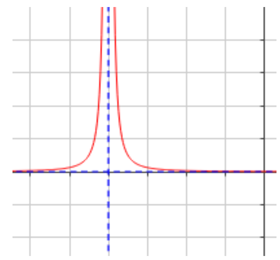
Domain: x≠ 0
Range: y≠ 0



Domain: x≠ 0
Range: y≠ 8



Domain x≠ 0
Range: y> 0



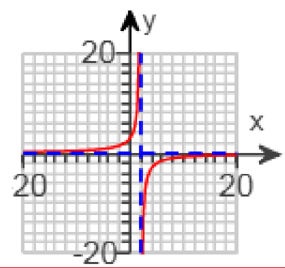
Domain: x≠ -8
Range: y> 0

$H(x) = \frac{-6}{x-2}$

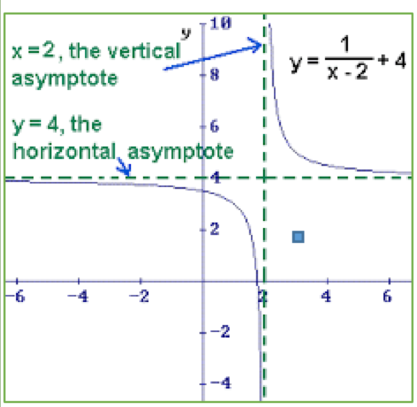
Flips across the x-axis and shifts right 2

Domain: x≠ 2
Range: y≠ 0

Vertical Asymptote x=2
Horizontal Asymptote y=0
No Oblique Asymptote



Asymptote Properties



Vertical asymptote: set denominator =0

***Or look at the horizontal shift**

Horizontal asymptote: y = 0 if $\frac{1}{x}$ (higher x is on bottom)

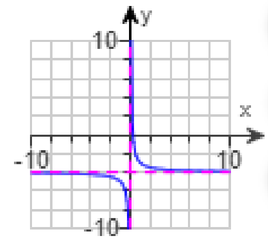
None if $\frac{x}{1}$ (higher x is on top)

Coefficient if $\frac{3x}{4x}$ y = $\frac{3}{4}$ (x exponents equal)

***Or look at the vertical shift**

16) For the function $F(x) = -4 + \frac{1}{x}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

(a) Which of the following transformations is required to graph the given function?



A. Shift the graph of $y = \frac{1}{x}$ down 4 units. -4 in front is down

The domain of the given function is $\{x|x \neq 0\}$. *red vertical line

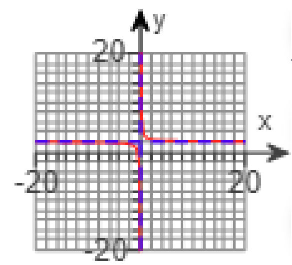
The range of the given function is $\{y|y \neq -4\}$. *red horizontal line

There is one vertical asymptote. It is $x = 0$. * equation of the red vertical line

There is one horizontal asymptote. It is $y = -4$. *equation of the red vertical line

There is no oblique asymptote.

17) For the function $F(x) = 2 + \frac{1}{x}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.



2 in front is up

(a) Choose the correct graph below.

The domain of the given function is $\{x|x \neq 0\}$. *red vertical line

The range of the given function is $\{y|y \neq -4\}$. *red horizontal line

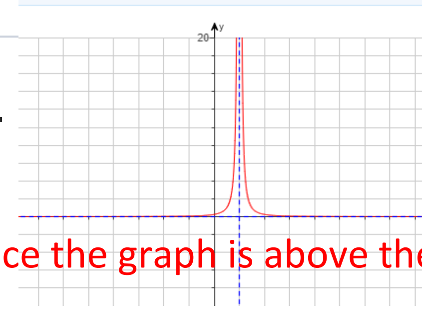
There is one vertical asymptote. It is $x = 0$. * equation of the red vertical line

There is one horizontal asymptote. It is $y = -4$. *equation of the red vertical line

There is no oblique asymptote.

18) For the function $F(x) = \frac{1}{(x-2)^2}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

(a) Choose the correct graph below.



The domain of the given function is $\{x|x \text{ is a real number}, x \neq 2\}$

The range of the given function is $\{y|y \text{ is a real number}, y > 0\}$

There is one vertical asymptote. It is $x = 2$.

There is one horizontal asymptote. It is $y = 0$

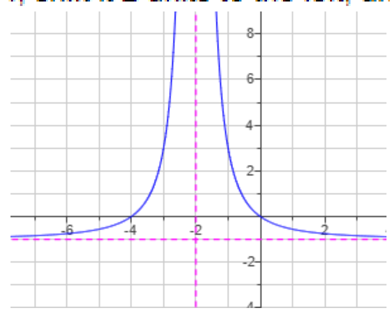
There is no oblique asymptote.

Notice the graph is above the x-axis

19) For the function $G(x) = -1 + \frac{4}{(x+2)^2}$, (a) graph the rational function using transformations, (b) use the final graph to find the domain and range, and (c) use the final graph to list any vertical, horizontal, or oblique asymptotes.

-1 in front is down, left 2

C. Vertically stretch the graph of $y = \frac{1}{x^2}$ by a factor of 4, shift it 2 units to the left, and 1 unit down.



Use the shifts and look at the pink asymptotes

The domain of the given function is $\{x \mid x \neq -2\}$.

The range of the given function is $\{y \mid y > -1\}$. Graph is above -1

There is one vertical asymptote. It is $x = -2$.

There is one horizontal asymptote. It is $y = -1$.

There is no oblique asymptote.

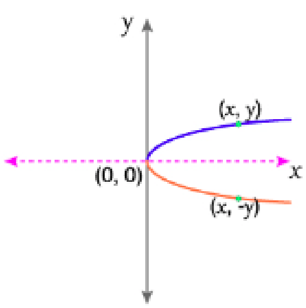
Horizontal asymptote:

$y = 0$ if $\frac{1}{x}$ (higher x is on bottom)

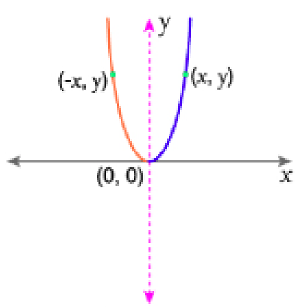
None if $\frac{x}{1}$ (higher x is on top)

Coefficient if $\frac{3x}{4x}$ $y = \frac{3}{4}$ (x exponents equal)

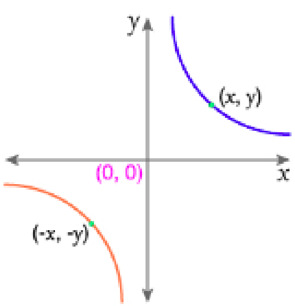
Symmetry in Graphs



x-axis symmetry



y-axis symmetry



origin symmetry

FINDING SYMMETRY ALGEBRAICALLY

$$\boxed{f(x) = \frac{4x}{x^2 - 16}}$$

Determine the symmetry by finding the value of $\boxed{f(-x)}$.

$$f(-x) = \frac{4(-x)}{(-x)^2 - 16}$$

$$= \frac{-4x}{x^2 - 16}$$

only top sign changes
ORIGIN SYMMETRY

$$\boxed{f(x) = \frac{6x^2}{x^2 - 4}}$$

Determine the symmetry by finding the value of $\boxed{f(-x)}$.

$$f(-x) = \frac{6(-x)^2}{(-x)^2 - 4}$$

$$= \frac{6x^2}{x^2 - 4}$$

Signs stay the same
Y-AXIS SYMMETRY

20) Follow the seven step strategy to graph the following rational function.

$$f(x) = \frac{2x}{x-4} \qquad f(-x) = \frac{-2x}{-x-4}$$

only top sign changes for ORIGIN SYMMETRY
signs stay the same Y-AXIS SYMMETRY

Neither y-axis symmetry nor origin symmetry

The y-intercept is 0 *set x = 0 $\frac{0}{-4} = 0$

The x-intercept is 0 *solve the numerator 2x=0, x=0

Find the vertical asymptote(s). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

*solve the denominator

A. The equation(s) of the vertical asymptote(s) is/are $x = 4$.

Find the horizontal asymptote(s). Select the correct choice below and, if necessary, fill in the answer box to complete your choice. Coefficient if $\frac{2x}{x}$ y = 2 (x exponents equal)

A. The equation(s) of the horizontal asymptote(s) is/are $y = 2$.

We are making an x|y table. Plug in x values to find the y values

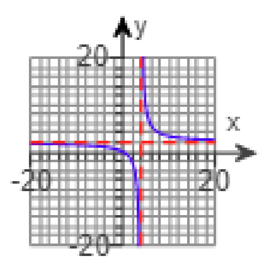
Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	−5	−4	1	6	7
$f(x) = \frac{2x}{x-4}$	$\frac{10}{9}$	1	$-\frac{2}{3}$	6	$\frac{14}{3}$

(Simplify your answer. Type an integer or a simplified fraction.)

Use the information obtained in the previous steps to graph the function between and beyond the vertical asymptotes. Choose the correct graph below.

look at asymptotes first then the points from the table if needed to choose the graph



only top sign changes for ORIGIN SYMMETRY
signs stay the same Y-AXIS SYMMETRY

21) Follow the seven step strategy to graph the following rational function.

$f(x) = \frac{4x}{x^2 - 16}$ $\quad f(-x) = \frac{-4x}{x^2-16}$ only top sign changes origin symmetry

The y-intercept is 0 *set x = 0

(x-4)(x+4) The x-intercept is 0 *solve the numerator

Vertical Asymptote x = - 4, x = 4 *solve the denominator

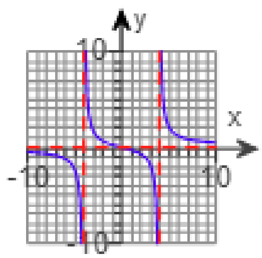
Horizontal Asymptote y = 0 y = 0 if $\frac{1}{x}$ (higher x is on bottom)

Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	−7	−5	−1	1	6	7
$f(x) = \frac{4x}{x^2 - 16}$	$-\frac{28}{33}$	$-\frac{20}{9}$	$\frac{4}{15}$	$-\frac{4}{15}$	$\frac{6}{5}$	$\frac{28}{33}$

(Simplify your answers.)

Use symmetry of origin



22) Follow the seven step strategy to graph the following rational function.

$f(x) = \frac{5x^2}{x^2 - 4}$ $\quad f(-x) = \frac{5x^2}{x^2-4}$ signs stay the same y-axis symmetry

The y-intercept is 0 *set x = 0

(x-2)(x+2) The x-intercept is 0 *solve the numerator

Vertical Asymptote x = - 2, x = 2 *solve the denominator

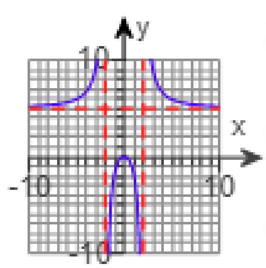
Horizontal Asymptote y = 5 Coefficient if $\frac{5x^2}{x^2}$ y = 5 (x exponents equal)

Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	−4	−3	$-\frac{1}{2}$	$\frac{1}{2}$	3	4
$f(x) = \frac{5x^2}{x^2 - 4}$	$\frac{20}{3}$	9	$-\frac{1}{3}$	$-\frac{1}{3}$	9	$\frac{20}{3}$

(Simplify your answers.)

Use symmetry of y-axis



23) Follow the seven step strategy to graph the following rational function.

$f(x) = \frac{-x}{x+7}$ $f(-x) = \frac{x}{-x+7}$ top and bottom signs change

Neither y-axis symmetry nor origin symmetry

The y-intercept is 0 *set x = 0

The x-intercept is 0 *solve the numerator

Vertical Asymptote x = - 7 *solve the denominator

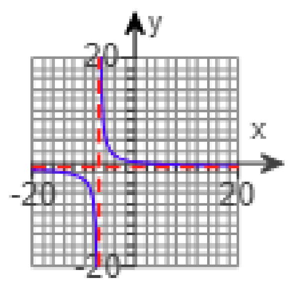
Horizontal Asymptote y = -1 **Coefficient if $\frac{-x}{x}$ y = -1 (x exponents equal)**

Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	-9	-8	$-\frac{1}{2}$	6	8
$\mathbf{f(x) = \frac{-x}{x+7}}$	$-\frac{9}{2}$	-8	$\frac{1}{13}$	$-\frac{6}{13}$	$-\frac{8}{15}$

(Simplify your answers.)

**look at asymptotes first then the points to choose the graph*



24) Follow the seven step strategy to graph the following rational function.

$f(x) = \frac{4}{x^2+2x-3}$ $f(-x) = \frac{4}{x^2-2x-3}$ top sign doesn't change

Neither y-axis symmetry nor origin symmetry

(x-1)(x+3) The y-intercept is $-\frac{4}{3}$ *set x = 0

The x-intercept is none *there is no x in the numerator

Vertical Asymptote x = 1, x = -3 *factor the denominator

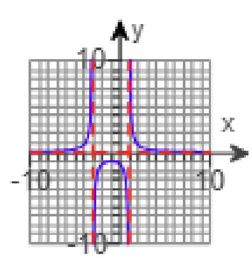
Horizontal Asymptote y = 0 **y = 0 if $\frac{1}{x}$ (higher x is on bottom)**

Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	-4	-1	0	4	6
$\mathbf{f(x) = \frac{4}{x^2+2x-3}}$	$\frac{4}{5}$	-1	$-\frac{4}{3}$	$\frac{4}{21}$	$\frac{4}{45}$

(Simplify your answers.)

Use symmetry of y-axis



25) Follow the seven step strategy to graph the following rational function.

$$f(x) = \frac{4x^2}{x^2+9}$$ $f(-x) = \frac{4x^2}{x^2+9}$ signs stay the same y-axis symmetry

The y-intercept is 0 *set x = 0

The x-intercept is 0 *solve the numerator

Vertical Asymptote none *can't factor the denominator

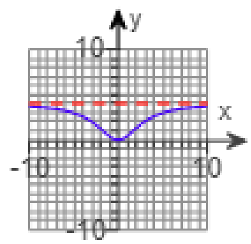
Horizontal Asymptote y = 4 **Coefficient if $\frac{4x^2}{x^2}$ y = 4 (x exponents equal)**

Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	-3	-1	1	4
$\mathbf{f(x) = \frac{4x^2}{x^2+9}}$	2	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{64}{25}$

(Simplify your answers.)

Use symmetry of y-axis



26) Follow the seven step strategy to graph the following rational function.

$$f(x) = \frac{x+2}{x^2+2x-8}$$ $f(-x) = \frac{-x+2}{x^2-2x-8}$ only one sign changes on top/not both

Neither y-axis symmetry nor origin symmetry

(x-2)(x+4) The y-intercept is $-\frac{1}{4}$ *set x = 0

The x-intercept is -2 *solve the numerator

Vertical Asymptote x = -4, x = 2 *factor the denominator

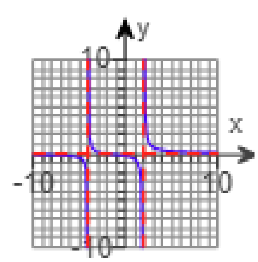
Horizontal Asymptote y = 0 **y = 0 if $\frac{1}{x}$ (higher x is on bottom)**

Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	-7	-6	-1	1	6	7
$\mathbf{f(x) = \frac{x+2}{x^2+2x-8}}$	$-\frac{5}{27}$	$-\frac{1}{4}$	$-\frac{1}{9}$	$-\frac{3}{5}$	$\frac{1}{5}$	$\frac{9}{55}$

(Simplify your answers.)

**look at asymptotes first then the points to choose the graph*



27) Follow the seven step strategy to graph the following rational function.

$$f(x) = \frac{x - 7}{x^2 - 49}$$

$f(-x) = \frac{-x-7}{x^2-49}$ only one sign changes on top/not both

Neither y-axis symmetry nor origin symmetry

The y-intercept is $\frac{1}{7}$ *set x = 0 $\frac{0-7}{0-49} = \frac{1}{7}$

The x-intercept is 7 *solve the numerator

To find vertical asymptote: $\textit{factored} = \frac{\cancel{x-7}}{\cancel{(x-7)}(x+7)} = \frac{1}{x+7}$

The bottom is vertical asymptote and the crossed-out term is a hole in the graph

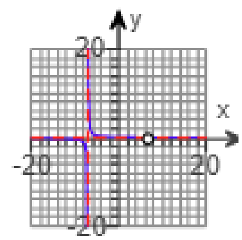
Vertical Asymptote x = -7 hole at x= 7

Horizontal Asymptote y = 0 **y = 0 if $\frac{1}{x}$ (higher x is on bottom)**

Plot points between and beyond each x-intercept and vertical asymptote. Find the value of the function at the given value of x.

x	−10	−9	6	8	9
$f(x) = \frac{x-7}{x^2-49}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{13}$	$\frac{1}{15}$	$\frac{1}{16}$

(Simplify your answers.)



look at asymptotes first then the points to choose the graph